

Mugberia Gangadhar Mahavidyalaya

Surprise Class Text Examination :: Mathematics:: B.Sc SEM-IV

Fourier Series: paper-CT9 (2019)

Total Marks : 26

Group-A: Answer any three questions $5 \times 3 = 15$

1. Expand the function $f(x) = e^x - 1$ in a Fourier series in $[0, 2\pi]$. Hence show that $\sum_{n=1}^{\infty} \frac{1}{n^2+1} = \frac{1}{2}[\pi \operatorname{cosech} \pi - 1]$.
2. If $f(x) = (\pi - |x|)^2$ on $[-\pi, \pi]$, prove that the Fourier series of f is given by

$$\frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}.$$

Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$.

3. Obtain the Fourier cosine series for the even function $f(x) = |x|$ on $[-\pi, \pi]$.
4. Expand the function $\sin cx$ on $[0, \pi]$ in a Fourier Cosine series, where c is not an integer.
5. Prove that for $0 \leq x \leq \pi$
 $x(\pi - x) = \frac{8}{\pi} \left(\frac{\sin x}{1^3} + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \dots \right)$. Hence deduce that
 $x = \frac{\pi}{2} - \frac{4}{\pi} \left(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \dots \right)$, $0 \leq x \leq \pi$.

Group-B: Answer any one question $5 \times 1 = 5$

1. State and Prove the Parseval's Identity.
2. If the series $\frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ converges uniformly to f on $[-\pi, \pi]$, then it is the Fourier series for f in $[-\pi, \pi]$.

Group-C: Answer any three questions $2 \times 3 = 6$

1. State the Fourier Series.
2. State the Dirichlet's Conditions
3. Examine if the trigonometric series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ is a Fourier series in $[-\pi, \pi]$.
4. Examine if the trigonometric series $\sum_{n=1}^{\infty} \frac{\cos nx}{\sqrt{n}}$ is a Fourier series in $[-\pi, \pi]$.